

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For any integer $N \geq 100$, let $T(N)$ be the count of twin prime pairs $(p, p+2)$ with $p \leq N$. Let $S_3(N)$ be the count of such pairs where the smaller prime p satisfies $p \bmod 3 = 1$. The conjecture states that the deviation of $S_3(N)$ from exactly half of $T(N)$ is bounded by the square root of the total count: $|S_3(N) - T(N)/2| < \sqrt{T(N)}$. A counterexample was discovered computationally: witness = $\{N: 50000, T(N): 705, S_3(N): 0, LHS: 352.5, RHS: 26.551836094703507\}$. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Twin prime conjecture — density analysis. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *For any integer $N \geq 100$, let $T(N)$ be the count of twin prime pairs $(p, p+2)$ with $p \leq N$. Let $S_3(N)$ be the count of such pairs where the smaller prime p satisfies $p \bmod 3 = 1$. The conjecture states that the deviation of $S_3(N)$ from exactly half of $T(N)$ is bounded by the square root of the total count: $|S_3(N) - T(N)/2| < \sqrt{T(N)}$. This asserts a 'square-root cancellation' in the distribution of twin primes across the two allowable residue classes modulo 3, analogous to probabilistic expectat*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$$witness = \{ 'N' : 50000, 'T(N)' : 705, 'S_3(N)' : 0, 'LHS' : 352.5, 'RHS' : 26.551836094703507 \}$$

Proof. Direct computation verifies that the witness $\{ 'N' : 50000, 'T(N)' : 705, 'S_3(N)' : 0, 'LHS' : 352.5, 'RHS' : 26.551836094703507 \}$ satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. $\square$

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.