A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: Let P_N be the set of primes of the form n^{}2+1 for 1 <= n <= N. Let A_N be the count of such primes where the generator n is itself a prime number. The conjecture states that for all N >= 1000, the ratio of the density of 'prime-generated' primes to . A counterexample was discovered computationally: witness = {'N': 50000, 'total_primes_form': 3613, 'prime_generated_count': 1, 'actual_ratio': 0.0002767783005812344, 'threshold': 0.09242333564642943}. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form n^{}2+1 — density and distribution. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *Let P_N be the set of primes of the form n^{}2+1 for 1 <= n <= N. Let A_N be the count of such primes where the generator n is itself a prime number. The conjecture states that for all N >= 1000, the ratio of the density of 'prime-generated' primes to the total density of such primes exceeds the reciprocal of the natural logarithm of N. Specifically: A_N*

/ |P_N| > 1 / ln(N). This suggests that prime generators are slightly over-represented among the solutions to n^{}2+1 being prime compared to the b

3 Counterexample

Theorem 1 (Falsification)**.** *The conjecture above is* ***false****. A counterexample is given by:*

$$witness = \{'N' : 50000, 'total\_primes\_form' : 3613, 'prime\_generated\_count' : 1, 'actual\_ratio' : 0$$

Proof. Direct computation verifies that the witness $\{'N' : 50000, 'total\_primes\_form' : 3613, 'prime\_generated\_count' : 1, 'actual\_ratio' : 0.0002767783005812344, 'threshold' : 0.09242333564642943\}$ satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. □

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.