

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For all $x \geq 1000$, the actual count of twin prime pairs up to x strictly exceeds the standard Hardy-Littlewood prediction ($2 \cdot C_2 \cdot x / \ln(x)^2$) but remains bounded above by the prediction augmented with a specific second-order correction term: $2 \cdot C_2 \cdot x / (\ln(x)^2 - 1.5 \cdot \ln(x))$. A counterexample was discovered computationally: witness = Upper bound failed: actual=705, pred_upper=654.6760580027285. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Twin prime density — Hardy-Littlewood conjecture verification. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For all $x \geq 1000$, the actual count of twin prime pairs up to x strictly exceeds the standard Hardy-Littlewood prediction ($2 \cdot C_2 \cdot x / \ln(x)^2$) but remains bounded above by the prediction augmented with a specific second-order correction term: $2 \cdot C_2 \cdot x / (\ln(x)^2 - 1.5 \cdot \ln(x))$. This implies the convergence to the asymptotic limit is approached from below with a predictable logarithmic lag.*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

witness = Upperboundfailed : actual = 705, pred_upper = 654.6760580027285

Proof. Direct computation verifies that the witness *Upperboundfailed : actual = 705, pred_upper = 654.6760580027285* satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.