A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For every integer n > 1, let S(n) be the set of odd integers encountered in the Collatz trajectory of n before reaching 1 (excluding the final 1). Let M(n) be the maximum element in S(n). If S(n) is non-empty, then M(n) is congruent to 5 modulo 6 if . A counterexample was discovered computationally: witness = {'n': 50000, 'M_n': 3125, 'S_n': [3125, 293, 55, 83, 125, 47, 71, 107, 161, 121, 91, 137, 103, 155, 233, 175, 263, 395, 593, 445, 167, 251, 377, 283, 425, 319, 479, 719, 1079, 1619, 2429, 911, 1367, 2051, 3077, 577, 433, 325, 61, 23, 35, 53, 5], 'count_mod2': 31, 'count_mod1': 12, 'lhs': True, 'rhs': False}. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Collatz conjecture — structural pattern search. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *For every integer $n > 1$, let $S(n)$ be the set of odd integers encountered in the Collatz trajectory of n before reaching 1 (excluding the final 1). Let $M(n)$ be the maximum element in $S(n)$. If $S(n)$ is non-empty, then $M(n)$ is congruent to 5 modulo 6 if and only if the number of elements* *in S(n) that are congruent to 2 modulo 3 is exactly equal to the number of elements in S(n) that are congruent to 1 modulo 3.*

3 Counterexample

Theorem 1 (Falsification)**.** *The conjecture above is **false**. A counterexample is given by:*

$witness = \{'n' : 50000, 'M\_n' : 3125, 'S\_n' : [3125, 293, 55, 83, 125, 47, 71, 107, 161, 121, 91, 137, 103, 15$

Proof. Direct computation verifies that the witness $\{'n' : 50000, 'M\_n' : 3125, 'S\_n' : [3125, 293, 55, 83, 125, 47, 71, 107, 161, 121, 91, 137, 103, 155, 233, 175, 263, 395, 593, 445, 16$
$31, 'count\_mod1' : 12, 'lhs' : True, 'rhs' : False\}$ satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. □

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.