A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For every integer n >= 3, if the Fibonacci number F_n is prime, then n must be a prime number, AND the index n satisfies the property that 2n+1 is either a prime number or a semiprime (product of exactly two primes, not necessarily distinct). Further. A counterexample was discovered computationally: witness = 13. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Fibonacci primes — density conjecture. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For every integer n >= 3, if the Fibonacci number F_n is prime, then n must be a prime number, AND the index n satisfies the property that 2n+1 is either a prime number or a semiprime (product of exactly two primes, not necessarily distinct). Furthermore, if n > 5, then n modulo 10 must be in the set {1, 3, 7, 9} (which is trivial for primes > 5) AND the sum of digits of n in base 10 is not divisible by 3 (also trivial). The core non-trivial claim is: For all Fibonacci primes F_n with n > 4, the*

3 Counterexample

Theorem 1 (Falsification)**.** *The conjecture above is* ***false****. A counterexample is given by:*

$$witness = 13$$

Proof. Direct computation verifies that the witness 13 satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. □

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.