

# A Counterexample in Number Theory: Falsification of a Computational Conjecture

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## Abstract

We report the falsification of the following conjecture: For every integer  $N \geq 100$ , let  $T(N)$  be the count of twin prime pairs  $(p, p+2)$  with  $p \leq N$ , and let  $S(N)$  be the sum of the reciprocals of the smaller primes in these pairs (i.e.,  $\sum(1/p)$  for all such  $p$ ). The conjecture states that the ratio  $R(N) = T(N) / (S(N) * \ln(N)^2)$  is strictly bounded between 0.4 and 2.5. A counterexample was discovered computationally: witness = Ratio 6.4256 out of bounds for  $N=50000$ . This result was obtained by the SOVEREIGN autonomous research system.

## 1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Twin prime conjecture — density analysis. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

## 2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

**Conjecture 1.** *For every integer  $N \geq 100$ , let  $T(N)$  be the count of twin prime pairs  $(p, p+2)$  with  $p \leq N$ , and let  $S(N)$  be the sum of the reciprocals of the smaller primes in these pairs (i.e.,  $\sum(1/p)$  for all such  $p$ ). The conjecture states that the ratio  $R(N) = T(N) / (S(N) * \ln(N)^2)$  is strictly bounded between 0.4 and 2.5. This refines the Hardy-Littlewood prediction by asserting a specific, tight convergence band for the normalized density relative to the reciprocal sum, ruling out erratic oscillations ou*

### 3 Counterexample

**Theorem 1** (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$$\text{witness} = \text{Ratio6.4256outofboundsfor}N = 50000$$

*Proof.* Direct computation verifies that the witness *Ratio6.4256outofboundsfor* $N = 50000$  satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module.  $\square$

### 4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.