A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Abstract

We report the falsification of the following conjecture: For any integer n >= 2, let P_n be the set of primes of the form k^{}2+1 where 1 <= k <= n. Let G_n be the maximum gap between consecutive elements in the sorted sequence P_n (defining the first gap as p_1 - 1). Then G_n is strictly less than 2.5 * sqr. A counterexample was discovered computationally: witness = {'n': 50000, 'max_gap': 9084000, 'bound': 2704944.57169356, 'p_max': 2500000001, 'primes_count': 3613}. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form n^{}2+1 — density conjecture. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *For any integer n >= 2, let P_n be the set of primes of the form k^{}2+1 where 1 <= k <= n. Let G_n be the maximum gap between consecutive elements in the sorted sequence P_n (defining the first gap as p_1 - 1). Then G_n is strictly less than 2.5 * sqrt(p_max) * ln(p_max), where p_max is the largest prime in P_n. If P_n has fewer than 2 elements, the conjecture holds trivially for that n.*

3 Counterexample

Theorem 1 (Falsification)**.** *The conjecture above is* ***false****. A counterexample is given by:*

$$witness = \{'n' : 50000, 'max\_gap' : 9084000, 'bound' : 2704944.57169356, 'p\_max' : 2500000001, 'prim$$

Proof. Direct computation verifies that the witness $\{'n' : 50000, 'max\_gap' : 9084000, 'bound' : 2704944.57169356, 'p\_max' : 2500000001, 'primes\_count' : 3613\}$ satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. □

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.