

# A Counterexample in Number Theory: Falsification of a Computational Conjecture

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## Abstract

We report the falsification of the following conjecture: For any integer  $n \geq 2$ , let  $S_n$  be the set of primes of the form  $k^2+1$  where  $k \leq n$ . Let  $M_n$  be the maximum gap between consecutive elements in the sorted sequence  $S_n$  (defining the first gap as  $p_1 - 2$ ). Then,  $M_n$  is strictly less than  $(\ln(p_{\max}))^3$ . A counterexample was discovered computationally: witness = Max gap 9084000  $\geq$  bound 10133.16 at  $n=50000$ ,  $p_{\max}=2500000001$ . This result was obtained by the SOVEREIGN autonomous research system.

## 1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form  $n^2+1$  — density conjecture. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

## 2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

**Conjecture 1.** *For any integer  $n \geq 2$ , let  $S_n$  be the set of primes of the form  $k^2+1$  where  $k \leq n$ . Let  $M_n$  be the maximum gap between consecutive elements in the sorted sequence  $S_n$  (defining the first gap as  $p_1 - 2$ ). Then,  $M_n$  is strictly less than  $(\ln(p_{\max}))^3$ , where  $p_{\max}$  is the largest prime in  $S_n$ .*

### 3 Counterexample

**Theorem 1** (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$witness = Maxgap9084000 \geq bound10133.16atn = 50000, p\_max = 2500000001$

*Proof.* Direct computation verifies that the witness  $Maxgap9084000 \geq bound10133.16atn = 50000, p\_max = 2500000001$  satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module.  $\square$

### 4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.