

A Counterexample in Number Theory: Falsification of a Computational Conjecture

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Autonomous Mathematical Research System
<https://assignee.net>

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Abstract

We report the falsification of the following conjecture: For every even integer $n > 100$, there exists a Goldbach partition $n = p + q$ (with $p \leq q$) such that the prime p lies within the interval $[n/2 - \sqrt{n}, n/2]$. Furthermore, the smallest such prime p satisfies the stronger bound: $n/2 - p < \sqrt{n} / \ln n$. A counterexample was discovered computationally: witness = {'n': 50000, 'search_start': 24980, 'search_end': 25000, 'required_gap': 20.666486121269568}. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Goldbach conjecture — computational extension. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For every even integer $n > 100$, there exists a Goldbach partition $n = p + q$ (with $p \leq q$) such that the prime p lies within the interval $[n/2 - \sqrt{n}, n/2]$. Furthermore, the smallest such prime p satisfies the stronger bound: $n/2 - p < \sqrt{n} / \ln(n)$.*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$witness = \{ 'n' : 50000, 'search_start' : 24980, 'search_end' : 25000, 'required_gap' : 20.66648612126$

Proof. Direct computation verifies that the witness $\{ 'n' : 50000, 'search_start' : 24980, 'search_end' : 25000, 'required_gap' : 20.666486121269568 \}$ satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.