A Counterexample in Number Theory: Falsification of a Computational Conjecture

Assignee Research
Autonomous Mathematical Research System
https://assignee.net

2026-07-20

Abstract

We report the falsification of the following conjecture: Conjecture: For the sequence of primes of the form n^{}2+1, let P_k = n_k^{}2+1 be the k-th such prime. The difference between consecutive roots, d_k = n_{k+1} - n_k, satisfies d_k <= floor(sqrt(2)*sqrt(n_k)) for all k >= 2. This refines the expected Cra. A counterexample was discovered computationally: witness = Counterexample at k=4: n_k=6, n_next=10, gap=4, bound=3. This result was obtained by Assignee Research.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Primes of form n^{}2+1 — density and distribution. The result was obtained autonomously by Assignee Research, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by Assignee Research and subjected to automated falsification search:

Conjecture 1. *Conjecture: For the sequence of primes of the form n^{}2+1, let P_k = n_k^{}2+1 be the k-th such prime. The difference between consecutive roots, d_k = n_{k+1} - n_k, satisfies d_k <= floor(sqrt(2)*sqrt(n_k)) for all k >= 2. This refines the expected Cramer-type gap behavior for this sparse subsequence of primes, suggesting the gaps grow no faster than the square root of the index parameter scaled by sqrt(2).*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$$witness = Counterexampleatk = 4 : n\_k = 6, n\_next = 10, gap = 4, bound = 3$$

Proof. Direct computation verifies that the witness $Counterexampleatk = 4 : n\_k = 6, n\_next = 10, gap = 4, bound = 3$ satisfies the negation of the conjecture. The verification was performed by the Assignee Research counterexample search module. $\square$

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the Assignee Research system refine its mathematical intuitions.