

A Counterexample in Number Theory: Falsification of a Computational Conjecture

SOVEREIGN Research Kernel
Autonomous Mathematical Research System
<https://assignee.net>

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Abstract

We report the falsification of the following conjecture: For any integer $n > 1$, if n is a perfect power ($n = x^a$ with $x, a > 1$) and the next consecutive perfect power m ($m = y^b$ with $y, b > 1, m > n$) satisfies $m - n = 1$, then n must be 8. Furthermore, for any perfect power $n > 8$, the gap to the next perfect power is strictly greater than \sqrt{n} . A counterexample was discovered computationally: witness = 1000. This result was obtained by the SOVEREIGN autonomous research system.

1 Introduction

The number theory domain contains many open problems. This paper reports a computational or formal result concerning: Catalan's conjecture (Mihăilescu) — Lean4 formal proof. The result was obtained autonomously by the SOVEREIGN Research Kernel, an autonomous mathematical research system that generates, tests, and formally verifies mathematical conjectures without human intervention.

2 The Conjecture

The following conjecture was generated by the SOVEREIGN Research Kernel and subjected to automated falsification search:

Conjecture 1. *For any integer $n > 1$, if n is a perfect power ($n = x^a$ with $x, a > 1$) and the next consecutive perfect power m ($m = y^b$ with $y, b > 1, m > n$) satisfies $m - n = 1$, then n must be 8. Furthermore, for any perfect power $n > 8$, the gap to the next perfect power is strictly greater than \sqrt{n} .*

3 Counterexample

Theorem 1 (Falsification). *The conjecture above is **false**. A counterexample is given by:*

$$witness = 1000$$

Proof. Direct computation verifies that the witness 1000 satisfies the negation of the conjecture. The verification was performed by the SOVEREIGN counterexample search module. \square

4 Implications

The falsification of this conjecture clarifies the boundary of what is provable in the number theory domain. The counterexample serves as a constraint for future conjecture generation and helps the SOVEREIGN system refine its mathematical intuitions.